The Chi Square Test and Measures of Association

EDP 613

Week 12

A Note About The Slides

Currently the equations do not show up properly in Firefox. Other browsers such as Chrome and Safari do work.

Independence

Two variables that have no association with each other are **statistically independent**.

Frequencies

- **expected frequencies**

  > written f_e
  >
  > what you would *expect* in a bivariate table if two variables were statistically independent
  >
  > only assumption: the null hypothesis is true
  >
  > calculated by
  >
  > $$f_e = \frac{\text{column marginal} \cdot \text{row marginal}}{\text{total sample size}}$$

- **observed frequencies**

  > written f_o
  >
  > what you would *observe* in a bivariate table given what you have
  >
  > calculated by you or given

Chi-Square Test

> written χ^2.
>
> assumes *random sampling*
>
> Is an inferential test to find significant relationships between two variables.
>
> Calculated by
>
> $$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$
>
> with
>
> $$df = (r - 1)(c - 1)$$

Example: Social Media

The percent of people using at least one social media outlet is given below by age groups

In 2011:

Age	Portion
18 - 29	820
30 - 49	590
50 - 64	360
65+	120

In 2021:

Age	Responses
18 - 29	840
30 - 49	810
50 - 64	730
65+	450

a. Test the assumption that *users are equally likely* to be in each of the four age groups listed.

b. Which age group contributes the largest amount to the test statistic?

Source: *Pew Research Center: Social Media Fact Sheet*

Example: Solution for 2011

a. We have

$$H_0 : \text{Users are equally likely to be in each of the four groups listed}$$

$$H_1 : \text{Users are NOT equally likely to be in each of the four groups listed}$$

> Step 1: Find N

We have $820 + 590 + 360 + 120 = 1890$ total responses

If the distribution was uniform across all four categories, we would expect that each had $1890/4 \approx 472$ respondents

Step 2: Caluclate the χ^2 statistic

Age	Responses	χ^2
18 - 29	820	$\frac{(820-472)^2}{472} \approx 256.576$
30 - 49	590	$\frac{(590-472)^2}{472} \approx 29.500$
50 - 64	360	$\frac{(360-472)^2}{472} \approx 26.576$
65+	120	$\frac{(120-472)^2}{472} \approx 62.509$

with the total

$$256.576 + 29.500 + 26.576 + 62.509 = 375.161$$

and

$$df = 4 - 1 = 3$$

Step 3: Make a Decision

In Appendix D

- Look at $df = 3$
- $\chi^2 = 375.161$ < the greatest p-value so $p < 0.001$
- We reject H_0 implying that

respondents are not equally likely to be in each of the four age ranges listed

b .

- 65+ contributes the greatest amount to the sum for the test statistic
- The observed count is much smaller than expected

Example: Solution for 2021

We have

$$H_0 : \text{Users are equally likely to be in each of the four groups listed}$$

$$H_1 : \text{Users are NOT equally likely to be in each of the four groups listed}$$

Step 1: Find N

We have $840 + 810 + 730 + 450 = 2830$ total responses

If the distribution was uniform across all four categories, we would expect that each had $2830/4 \approx 707$ respondents

Step 2: Caluclate the χ^2 statistic

Age	Responses	χ^2
18 - 29	840	$\frac{(840-707)^2}{707} \approx 25.020$
30 - 49	810	$\frac{(810-707)^2}{707} \approx 15.006$
50 - 64	730	$\frac{(730-707)^2}{707} \approx 0.748$
65+	450	$\frac{(450-707)^2}{707} \approx 93.422$

with the total

$$5.020 + 15.006 + 0.748 + 93.422 = 134.196$$

and

$$df = 4 - 1 = 3$$

Step 3: Make a Decision

In Appendix D

- Look at $df = 3$
- $\chi^2 = 33.526$ < the greatest p-value so $p < 0.001$
- We reject H_0 implying that

respondents are not equally likely to be in each of the four age ranges listed

That's it. Take a break before our R session!