Estimations

EDP 613

Week 8

A Note About The Slides

Currently the equations may not show up properly in Firefox. Other browsers such as Chrome and Safari do appear to render them correctly.

A Note About Probability

We're going to introduce some concepts from Chapter 8 here.

Statistical Methods I

From To

Descriptive Statistics

mathematical techniques for organizing and summarizing a set of numerical data

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Inferential Statistics

generalizing from a sample to a population

Terms

- **Statistic** - Mathematical expression that describes some aspects of a set of scores for a sample
- **Parameter** - Describes some aspect of a set of scores for a population

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First a Brief Intro to Hypothesis Testing

> - Formally - Testing an assumption about a population parameter
> - In Better Terms - An assumption about a particular situation of the world that is testable

The Null Hypothesis

- Represented as H_0
- is basically what you expect to happen before you run an experiment
- *You have to know what the Null is!*

The Alternative Hypothesis

- Represented as H_1 (or H_A)
- is basically what else could happen if what you expect doesn't occur
- *You don't have to know this!*

Tests of Statistical Significance

- *Formally* - Done to determine whether H_0 or H_1 can be rejected
- *Better Explanation* - Test to figure out whether you can reasonably say if your initial assumption won't happen
- *Results* - If the outcomes of a study don't go against what you expected to happen, then you aren't finding anything new or surprising

Term

A **(statistical) estimation** is a sample statistic is used to estimate the value of an unknown population parameter.

Idea of Positive and Negative Outcomes

- The Null hypothesis H_0 is typically assuming nothing is going to happen
  - If H_0 turns out to be right, then its called a ***negative*** outcome because nothing changed.
  - If H_1 turns out to be right, then its called a ***positive*** outcome because something that you expected to happen didn't happen.

> - Experiment: Over the span of one year, a group of people with ADHD gets an experimental pill that may help them focus better than their current medication
>
> > - H_0: Group stays the same (expected)
> > - H_A: Group is more focused (what we want to happen)
>
> - Results: After an assessment
>
> > - if the Group doesn't show greater focus, then we have a ***negative*** outcome because that's what was expected to happen
> > - if the Group shows greater focus, then we have a ***positive*** outcome because that's NOT what was expected to happen

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Notes about H_0 and H_A

H_A is typically not the only alternative explanation

- What if the Group was found to more focused?

> - As a rule of thumb don't say that H_A is correct unless you absolutely know there are two outcomes (aka *binary outcomes*)
> - Instead write that "we reject H_0" because you don't know if that's the ONLY alternative hypothesis.
>
> > - It could also be that in other experiments that groups are found to be less focused!

- What if nothing happened to the Group?

> - You can absolutely say that H_0 is correct because that's what you expected
> - So you can write that "we accept H_0"

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Formal Table of Statistical Error Types

Decision	Null is True	Null is False
Reject Null	**Type I Error** (aka ***False Positive***)	Correct Outcome (aka *True Positive*)
Fail to Reject Null	Correct Outcome (aka *True Negative*)	**Type II Error** (aka ***False Negative***)

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Nutshell Table of Statistical Error Types

Decision	Your first thought was right	Your first thought was wrong
You changed your mind	**You changed your mind BUT the reality is you shouldn't have**	You changed your mind AND in reality that was the right decision
Results in a	***False Positive / Type I Error***	*True Positive*
You didn't change your mind	You didn't change your mind AND in reality that was the right decision	**You didn't change your mind BUT the reality is that you should have**
Results in a	*True Negative*	***False Negative / Type II Error***

Example

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Alpha

Formally

> - rejecting H_0 when it is true
> - the probability of making a **Type I Error**

In Better Terms

> - the chance of making the wrong decision when what was initially expected to happen actually occurs
> - Given by α
> - Ranges from 0-1 like all other probabilities

Typically $\alpha = 0.05$ but its really context dependent

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Example

For airplanes

- if they fly people around, then when **analyzing failures**
  > - you may want to lower the probability of making a wrong decision
  > - use a **smaller** α
- if they're made of paper, then when **analyzing failures**
  > - you might be willing accept the higher risk of making the wrong decision
  > - use a **higher** α

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Beta

Formally

> - not rejecting the H_0 when H_1 is true
> - the probability of making a **Type II Error**

In Better Terms

> - the chance of making the wrong decision when an something else actually occurs
> - Given by β
> - Ranges from 0-1 like all other probabilities

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Power

- $1-\beta$ is called **statistical power**
- extremely important!
- Formally - the probability of NOT making a Type II error
- In Better Terms - the chance that you can separate if an outcome is a result of something occurring vs. pure luck!

Decision Making

Reality	Rejected Null	Did Not Reject Null
H_0 is true	Type I Error α Chance of rejecting H_0 when it is true / ***Level of Significance***	Correct decision $1 - \alpha$ ***Level of Confidence***
H_0 is false	Correct Decision $1 - \beta$ ***Statistical Power!***	Type II Error β ***Rate of a Type II Error*** / Chance of accepting H_0 when it is false

Decision Making

Null	$H_0 =$	"Forecast says its NOT going to rain"
Alternative	$H_1 =$	"Something else will happen"

Reality	Did not reject the forecast	Rejected forecast
Forecast was right	Did not take an umbrella and you're dry	Took an umbrella AND you're dry but may look silly or possibly fancy
Forecast was wrong	Did not take an umbrella AND you're wet	Took an umbrella AND you're dry

Note: You could have also gotten wet from snow, a flood, etc. so again ***the alternative hypothesis generally does not imply the opposite!***

Estimation

- **(Statistical) Estimation** - a sample statistic is used to estimate the value of an unknown population parameter
  - **Point estimation** - use of sample data to calculate a single value
  - **Interval estimation** - use of sample data to calculate a possible range of values

Selecting a sample mean

Classification	Hypothesis Testing	Point/Interval Estimation
Process	Determine the probability of getting that mean if the Null is true	Estimate the value of a population mean
Outcomes	Gain information about the population mean	Gain information about the population mean

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Updating Estimation for Sample Means

- **Point estimation** - use of sample data to calculate a single **mean** value
    - Benefit - the sample mean will equal the population mean on average
    - Drawback - unable to figure out if a sample mean actually equals the population mean
- **Interval estimation** - use of sample data to calculate a possible range of **mean** values

The Characteristic of Hypothesis Testing and Estimation

Question	Hypothesis Testing	Point/Interval Estimation
Do we know the population mean?	Yes its the Null hypothesis	No we're trying to estimate it
What is the process use dto determine?	The chance of obtaining a sample mean	The value of a population mean
What is learned?	Whether the population mean is likely correct	The range of values within which the population mean is probably contained
What is our decision?	To retain or reject the null hypothesis	No actual decison

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Confidence

- **Confidence Interval** - an interval that contains an unknown parameter (e.g. μ) with certain degree of confidence

- **Level of Confidence** - probability or likelihood that an interval estimate will contain an unknown population parameter

Determining the Confidence Interval

1. Calculate the standard error of the mean

$$\sigma_{\overline{Y}} = \frac{\sigma}{\sqrt{N}}$$

2. Decide on a level of confidence

Probability	**z-score**
0.90	1.645
0.95	1.96
0.99	2.576

Again its typical to have a 95% level of confidence thereby making

$$\alpha = 0.05$$

Determining the Confidence Interval (continued)

3. $CI = \overline{Y} \pm z \cdot \sigma_{\overline{Y}}$

4. Interpret the results

Example

IQ scores in the general healthy population are approximately normally distributed with 100 ± 15. In a sample of 100 students a sample mean IQ of 103. Find the 90% confidence interval for this data.

Firstly we have $N = 100$, $\mu = 100$, $\sigma = 15$, and $\overline{Y} = 103$.

1. $$\sigma_{\overline{Y}} = \frac{\sigma}{\sqrt{N}} = \frac{15}{\sqrt{100}} = 1.50$$

2. Want to find 90% confidence interval, so choose a 90% level of confidence.

$$z \cdot \sigma_{\overline{Y}} = 1.645 \cdot 1.50 = 2.47$$

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3. So

$$90\%\, CI = 103 \pm 2.47 = (105.47, 100.53)$$

4. We are 90% confident that the overall mean IQ is between 100.53 and 105.47.

That's it. Take a break before our R session!