# Testing Hypotheses

## EDP 613

### Week 9

# A Note About The Slides

Currently the equations do not show up properly in Firefox. Other browsers such as Chrome and Safari do work

# A Note About Previous Items

We're going to use some introduce some concepts from Chapter 7 here as well

# Definitions

The **margin of error** (**MoE**) is

> - *Formally* : the range of values above and below a sample statistic within a confidence interval
> - *In a nutshell* : how many percentage points your results will differ from the real population value
> - NOT the same as a **confidence interval**

(Statistical) **significance** is

> - *Formally* : a measure of the probability of the null hypothesis being true compared to the acceptable level of uncertainty regarding the true answer
> - *In a nutshell* : a result is probably not due to chance so it is likely real
> - NOT the same as a **practical significance**

# Interpretation

Example result:

A **95%** confidence interval with a
**3% margin of error**

What it means:

Your statistic will be within
**3 percentage points** of the real population value
**95%** of the time

The **MoE** is a probability!

# Back to Hypothesis Testing

Recall

The **null hypothesis** $H_0$ states

> - *Formally* : that a parameter is equal to a specific value
> - *Informally* : nothing probably happened

The **alternative hypothesis** $H_1$ states

> - *Formally* : that a parameter differs from the value specified by the null hypothesis
> - *Informally* : something probably happened

# More about Hypothesis Testing

Say a null hypothesis is $H_0\colon \mu = 50$. Then three things can occur from a frequentist perspective

- $H_1 < 50$: alternative hypothesis states that the parameter is *less* than the value of the null.

  > - You know to test for everything to the **left** of $H_1 = 50$.
  > - Called a **left-tailed test**

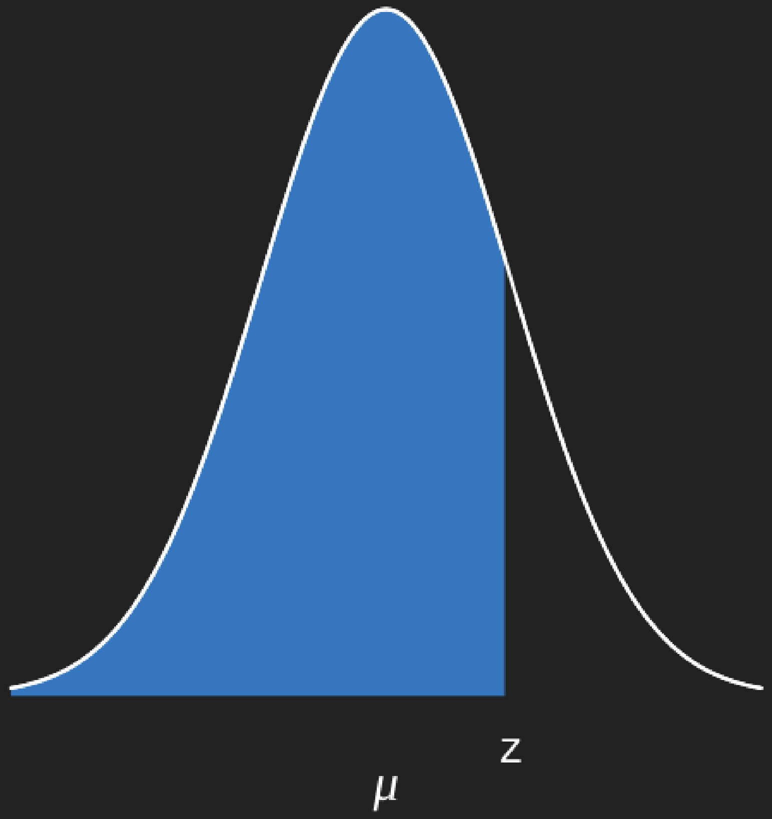
- $H_1 > 50$: alternative hypothesis states that the parameter is *more* than the value of the null.

  > - You know to test for everything to the **right** of $H_1 = 50$.
  > - Called a **right-tailed test**

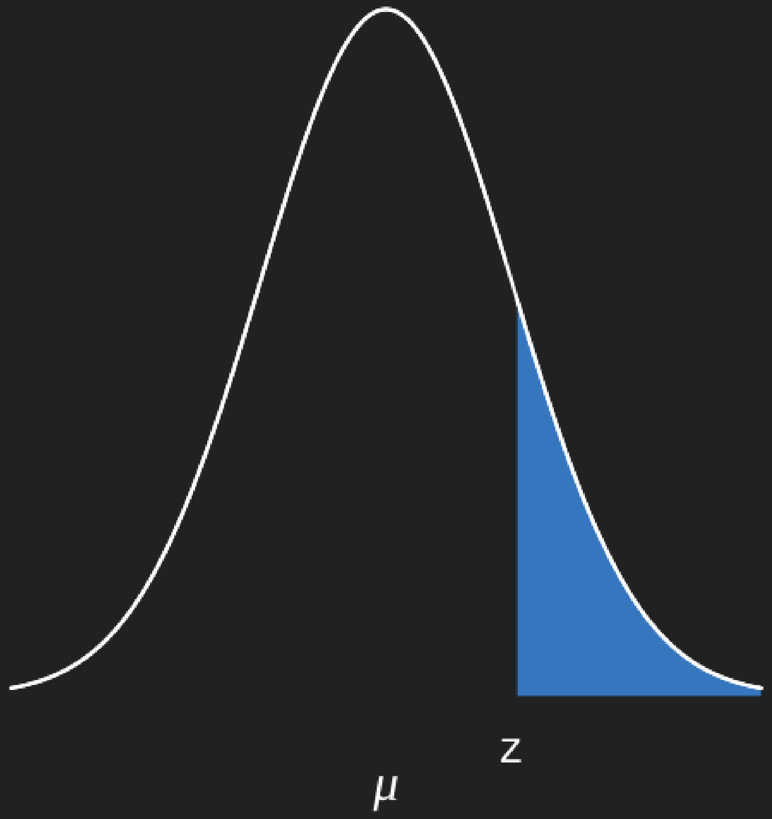
- $H_1 \neq 50$: alternative hypothesis states that the parameter is *note* the value of the null.

  > - You know to test for everything to the **left** and **right** of $H_1 = 50$.
  > - Called a **two-tailed test**

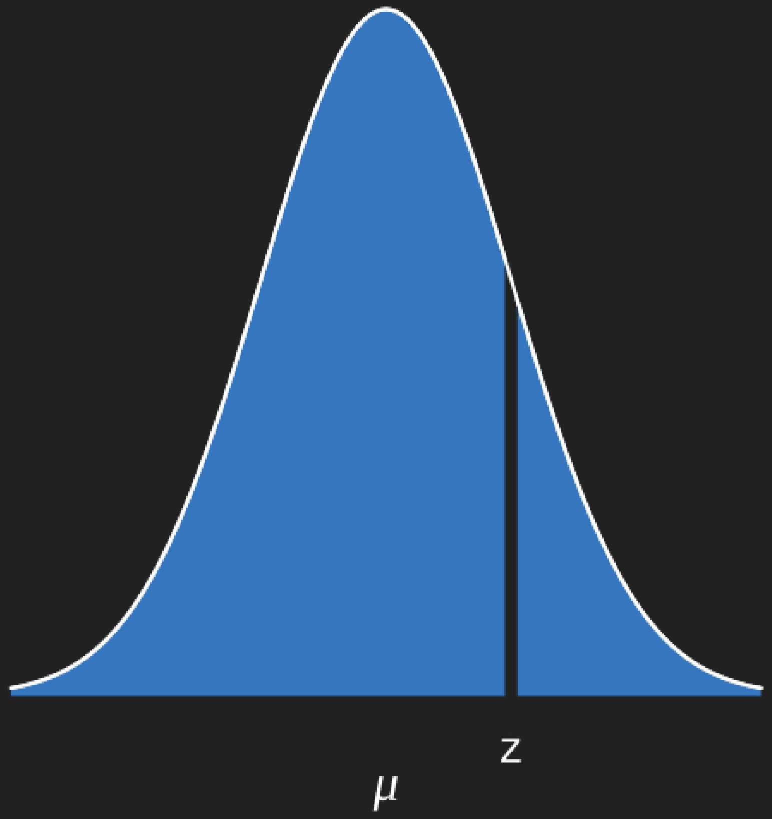
# $H_1 < H_0$: Left-Tailed Test



# $H_1 > H_0$: Right-Tailed Test



# $H_1 \neq H_0$: Two-Tailed Test



# Testing Methods

**Critical Value ($CV$)**

$p$ - **value**

# Critical Value

- Uses a **test statistic** - determines how strong the disagreement between a sample mean and a null hypothesis
- Idea: We should reject $H_0$ if the value of the test statistic is unusual when we assume $H_0$ to be true
- Process
  - We choose a *CV* which forms a boundary between values that are considered unusual and values that are not
  - The region containing the unusual values is called the **critical region**
  - If the value of the test statistic is in the critical regions, we *reject* $H_0$

# Transitioning (Again!)

Going from a $z$-distribution

*known population variance using $z$-scores*

to

a $t$-distribution

*known sample variance using $t$-tests*

# Note

We have to assume that for a large enough sample size, the $t$-distribution will closely match, or estimate, the a $z$-distribution

# Things to note

- $t$-distribution table can be located in Appendix C
- Assumptions
  - *Normality* - Samples are drawn from a population that fits a bell curve
  - *Independence* - Samples do not share values
  - *Random Sampling* - Samples are randomized
  - ***Homogeneity*** (for more than one sample) - Samples have a similar makeup

# Steps to Solving

1. *Interpret the Question into Layman's Terms*
2. *Set Acceptable Threshold of Committing a Type I Error*
3. *State the Research Hypothesis*
4. *Calculate the Test Statistic*
5. *Determine the Critical Value*
6. *State the Decision Rule*
7. *Interpret the Results*

# One-sample $t$-test

- allows us to determine whether the mean of a sample data set is different than a known value
- used when the population variance is not known
- can be used when the sample size is small ~ typically $N < 30$.

# What the heck is a *degree of freedom*?

1. Forget statistics
2. Say you only own seven hats and want to wear a different one each day of the week.
3. Process
    - Day 1: Choose from 7
    - Day 2: Choose from 6

      $\vdots$

      $\vdots$
    - Day 6: Choose from 2
    - Day 7: Choose from 1

   You had 7 - 1 = **6** *days of hat freedom!*

That's essentially a degree of freedom (written *df*)

# One Sample Mean

| *t*-distribution | *with degrees of freedom* |
| --- | --- |
| $t = \frac{\overline{Y} - \mu}{s/\sqrt{N}}$ | $df = N - 1$ |

# Example

Is the the median household income of West Virginia counties different than the national average?

| Region | Median Household Income | Median Male Salary | Median Female Salary | Population | Households |
|---|---|---|---|---|---|
| United States | $68,703 | $57,511 | $43,820 | 331,449,281 | 139,684,244 |
| West Virginia | $48,850 | $57,456 | $47,299 | 1,793,716 | 763,831 |

# *Interpret the Question into Layman's Terms*



From **Median Household Income**, we think WV (48,850 USD) is *practically lower* than the US (68,850 USD), but is it *significantly lower*?

# Set Acceptable Threshold of Committing a Type I Error

$\alpha = 0.5$

# *State the Research Hypothesis*

$H_0$

The **Median Household Income** of West Virginia counties is NOT significantly less than the national average

$H_1$

The **Median Household Income** of West Virginia counties is significantly less than the national average

# *Calculate the Test Statistic* (1/2)

Population

$$\mu = \$68,703$$

Sample

$$N = 61$$

$$s = \$5075.28$$

$$\overline{Y} = \$45732.20$$

*df*

$$61 - 1 = 60$$

# Calculate the Test Statistic (2/2)

$$
\begin{aligned}
t &= \frac{45732.20 - 68703.00}{5075.28/\sqrt{61}} \\
&\approx -0.579
\end{aligned}
$$

# Determine the Critical Value

In Appendix C, we see that for $df = 60$ at $\alpha = 0.05$, we have $t_{crit} = 1.671$

# State the Decision Rule

Since $-0.579 < 1.671$ we reject $H_0$

# *Interpret the Results*

*The **Median Household Income** is significantly less than the national average!*

# Two-sample Mean

Used to compare one sample mean to another.

- We use two different test:
  - Equal variances
  - Unequal variances (assumed)
- **Homoscedasticity** – the assumption of equal variances.

# Difference Between Two Independent Means

- Observations in each sample are not related
- Need to compare differences between the sample means

Estimated Standard Error

$$S_{\overline{Y_1}-\overline{Y_1}} = \sqrt{\frac{(N_1-1)\cdot s_1^2+(N_2-1)\cdot s_2^2}{(N_1+N_2)-2}} \cdot \sqrt{\frac{N_1+N_2}{N_1 \cdot N_2}}$$

difference between means *t*-statistic

$$t = \frac{\overline{Y_1}-\overline{Y_2}}{S_{\overline{Y_1}-\overline{Y_1}}}$$

*df*

$$df = (N_1+N_2)-2$$

# Difference Between Proportions

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Estimated Standard Error

$$S_{p_1-p_2} = \sqrt{\frac{p_1(1-p_1)}{N_1} + \frac{p_2(1-p_2)}{N_2}}$$

difference between means *z*-statistic

$$z = \frac{p_1 - p_2}{S_{p_1-p_2}}$$

# Example

As part of the Pew Internet and American Life Project, researchers conducted two surveys in late 2009. The first survey asked a random sample of 800 U.S. teens about their use of social media and the Internet. A second survey posed similar questions to a random sample of 2253 U.S. adults. In these two studies, 73% of teens and 47% of adults said that they use social-networking sites. Use these results to construct and interpret a 95% confidence interval for the difference between the proportion of all U.S. teens and adults who use social-networking sites.

# *Interpret the Question into Layman's Terms*

Is there a difference between the percent of teens and adults who use social networking sites?

# Set Acceptable Threshold of Committing a Type I Error

$\alpha = 0.05$

# State the Research Hypothesis

$H_0$

There is no difference between the proportion of teens and adults who use social media

$H_1$

There is a difference between the proportion of teens and adults who use social media

# Calculate the Test Statistic

Statistical Methods I

$$S_{p_1-p_2} = \sqrt{\frac{0.73(1-0.73)}{800} + \frac{0.47(1-0.47)}{2253}}$$

$$\approx 0.0189$$

$$z = \frac{0.73 - 0.47}{0.0189}$$

$$\approx 13.76$$

# State the Decision Rule

$z = 13.76$ is greater than $p$ value implying that we reject $H_0$

# *Interpret the Results*

We are 95% confident that in late 2009 more teens than adults in the United States engaged in social media

# That's it. Take a break before our R session!